CHAPTER 1

1.1 $\rho = \frac{p}{RT} = \frac{(5.6)(2116)}{(1716)(850)} = \boxed{0.00812 \text{ slug/ft}^3}$

$v = \frac{1}{\rho} = \boxed{123 \text{ ft}^3\text{/slug}}$

1.2 $\rho = \frac{p}{RT} = \frac{(10)(1.01 \times 10^5)}{(287)(320)} = \boxed{11.0 \text{ kg/m}^3}$

$n = \frac{p}{RT} = \frac{(10)(1.01 \times 10^5)}{(1.38 \times 10^{-23})(320)} = \boxed{2.87 \times 10^{26}\text{/m}^3}$

$\eta = \frac{pv}{RT} = \frac{p}{\rho RT} = \frac{(10)(1.01 \times 10^5)}{(11.0)(8314)(320)} = 0.0345 \frac{\text{kg} - \text{mole}}{\text{kg}}$

1.3 From the definition of enthalpy,

$$h = e + p\,v = e + RT \qquad \text{(A1)}$$

For a calorically perfect gas, this becomes

$c_p\,T = c_v\,T + RT$, or $\boxed{c_p - c_v = R}$

For a thermally perfect gas, Eq. (A1) is first differentiated

$dh = de + Rdt$

or,

$c_p\,dT = c_v\,dT = Rdt$

or,

$\boxed{c_p - c_v = R}$

1.4 $s_2 - s_1 = c_p \ \ell n \frac{T_2}{T_1} - R \, \ell n \ \frac{p_2}{p_1}$

(a) R = 1716 ft-lb/slug°R

$$c_p = \frac{\gamma R}{\gamma - 1} = \frac{(1.4)(1716)}{0.4} = 6006 \text{ ft-lb/slug°R}$$

$s_2 - s_1 = (6006) \ \ell n \ (1.687) - (1716) \ \ell n \ 4.5$

$s_2 - s_1 = \boxed{559.9 \text{ ft-lb/slug°R}}$

(b) R = 287 joule/kg°K

$$c_p = \frac{\gamma R}{\gamma - 1} = \frac{(1.4)(287)}{0.4} = 1004.5 \text{ joule/kg°K}$$

$s_2 - s_1 = (1004.5) \ \ell n \ (1.687) - 287 \ \ell n \ (4.5)$

$s_2 - s_1 = \boxed{93.6 \text{ joule/kg°K}}$

1.5 $\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$

$$p_2 = p_1 \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = 1800 \ (400/500)^{\frac{1.4}{0.4}}$$

$p_2 = \boxed{824.3 \text{ lb/ft}_2}$

$$\rho_2 = \frac{p_2}{RT_2} = \frac{824.3}{(1716)(400)} = \boxed{0.0012 \text{ slug/ft}^3}$$

1.6 Volume of room = (20)(15)(8) = 2400 ft^3

Standard sea level density = 0.002377 slug/ft^3

Mass of air = (0.002377)(2400) = $\boxed{5.70 \text{ slug}}$

Weight = Mass x acceleration of gravity = (5.7)(32.2) = $\boxed{184 \text{ lb}}$

1.7

(a) $dp = -\rho V dV$

and $d\rho = \rho\,\tau\,dp$, or $dp = \dfrac{d\rho}{\rho\tau}$

Combining:

$$\frac{d\rho}{\rho\tau} = -\rho\,VdV$$

$$d\rho = -\,\tau\,p^2\,VdV$$

$$\frac{d\rho}{\rho} = -\,\tau\,\rho\,VdV$$

$$\frac{d\rho}{\rho} = -\,\tau\,\rho\,V^2\,\frac{dV}{V}$$

(b) $\tau_s = \dfrac{1}{\gamma p} = \dfrac{1}{(1.4)(1.01\ \text{x}\ 10^5)} = 7.07 \text{ x } 10^{-6}\ m^2/N$

$$\frac{d\rho}{\rho} = \tau_s\,\rho\,V^2\,\frac{dV}{V} = -\,(7.07 \text{ x } 10^{-6})(1.23)(10)^2(0.01)$$

$$\frac{d\rho}{\rho} = \boxed{-\,8.7 \text{ x } 10^{-6}}$$

(c) Here, $\dfrac{d\rho}{\rho}$ will be larger by the ratio $\left(\dfrac{1000}{10}\right)^2$.

$$\frac{d\rho}{\rho} = (-\,8.7 \text{ x } 10^{-6})\left(\frac{1000}{10}\right)^2 = \boxed{-\,8.7 \text{ x } 10^{-2}}$$

<u>Comment</u>: By increasing the velocity of a factor of 100, the fractional change in density is increased by factor of 10^4. This is just another indication of why high-speed flows must be treated as compressible.

CHAPTER 2

2.1 Consider a two-dimensional body in a flow, as sketched in Figure A. A control volume is drawn around this body, as given in the dashed lines in Figure A. The control volume is bounded by:

1. The upper and lower streamlines far above and below the body (ab and hi, respectively.)
2. Lines perpendicular to the flow velocity far ahead of and behind the body (ai and bh, respectively).
3. A cut that surrounds and wraps the surface of the body (cdefg).

The entire control volume is abcdefhia. The width of the control volume in the z direction (perpendicular to the page) is unity. Stations 1 and 2 are inflow and outflow stations, respectively.

Assume that the contour abhi is far enough from the body such that the pressure is everywhere the same on abhi and equal to the freestream pressure $p = p_\infty$. Also, assume that the inflow velocity u_1 is uniform across ai (as it would be in a freestream, or a test section of a wind tunnel.) The outflow velocity u_2 is not uniform across bh, because the presence of the body has created a wake at the outflow station. However, assume that both u_1 and u_2 are in the x direction; hence, u_1 = constant and $u_2 = f(y)$.

Consider the surface forces on the control volume shown in Figure A. They stem from two contributions:

1. The pressure distribution over the surface, abhi,

$$-\iint_{abhi} p \, d\mathbf{S}$$

2. The surface force on def created by the presence of the body

Figure A

The surface shear stress on ab and hi has been neglected. Also, note that in Figure A the cuts cd and fg are taken adjacent to each other; hence any shear stress or pressure distribution on one is equal and opposite to that on the other; i.e., the surface forces on cd and fg cancel each other. Also, note that the surface on def is the <u>equal and opposite reaction</u> to the shear stress and pressure distribution created by the flow over the surface of the body. To see this more clearly, examine Figure B. On the left is shown the flow over the body. The moving fluid exerts pressure and shear stress distributions over the body surface which create a resultant aerodynamic force per unit span **R′** on the body. In turn, by Newton's third law, the body exerts equal and opposite pressure and shear stress distributions on the flow, i.e., on the part of the control surface bounded by def. Hence, the body exerts a force **–R′** on the control surface, as shown on the right of Figure B. With the above in mind, the total surface force on the entire control volume is

$$\text{Surface force} = -\iint_{abhi} p\, dS - R' \qquad (1)$$

Moreover, this is the total force on the control volume shown in Figure A because the volumetric body force is negligible.

Consider the integral form of the momentum equation as given by Equation (2.11) in the text. The right-hand side of this equation is physically the force on the fluid moving through the control volume. For the control volume in Figure A, this force is simply the expression given by Equation (1). Hence, using Equation (2.11), with the right-hand side given by Equation (1), we have

$$\frac{\partial}{\partial t} \oiiint_{\upsilon} \rho \, \mathbf{V} \frac{dV}{V} \, d\nu + \oiint_{S} (\rho \mathbf{V} \cdot \mathbf{dS}) \, \mathbf{V} = -\iint_{abhi} p \, \mathbf{DS} - \mathbf{R}' \tag{2}$$

Figure B

Assuming steady flow, Equation (2) becomes

$$R' = - \oiint_{S} (\rho \mathbf{V} \cdot \mathbf{dS}) \, \mathbf{V} - \iint_{abhi} p \, \mathbf{dS} \tag{3}$$

Equation (3) is a vector equation. Consider again the control volume in Figure A. Take the x component of Equation (3), nothing that the inflow and outflow velocities u_1 and u_2 are in the x direction and the x component of R′ is the aerodynamic drag per unit span D′:

$$D' = -\oiint_S (\rho \mathbf{V} \cdot \mathbf{dS})\, u - \iint_{abhi} (p\, dS)_x \qquad (4)$$

In Equation (4), the last term is the component of the pressure force in the x direction. [The expression $(p\ dS)_x$ is the x component of the pressure force exerted on the elemental area dS of the control surface.] Recall that the boundaries of the control volume abhi are chosen far enough from the body such that p is constant along these boundaries. For a constant pressure.

$$\iint_{abhi} (p\, dS)_x = 0 \qquad (5)$$

because, looking along the x direction in Figure A, the pressure force on abhi pushing toward the right exactly balances the pressure force pushing toward the left. This is true no matter what the shape of abhi is, as long as p is constant along the surface. Therefore, substituting Equation (5) into (4), we obtain

$$D' = -\oiint_S (\rho \mathbf{V} \cdot \mathbf{dS})\, u \qquad (6)$$

Evaluating the surface integral in Equation (6), we note from Figure A that:

1. The sections ab, hi and def are streamlines of the flow. Since by definition **V** is parallel to the streamlines and **dS** is perpendicular to the control surface, along these sections **V** and **dS** are perpendicular vectors, and hence $\mathbf{V} \cdot \mathbf{dS} = 0$. As a result, the contributions of ab, hi and def to the integral in Equation (6) are zero.

2. The cuts cd and fg are adjacent to each other. The mass flux out of one is identically the mass flux into the other. Hence, the contributions of cd and fg to the integral in Equation (6) cancel each other.

As a result, the only contributions to the integral in Equation (6) come from sections ai and bh. These sections are oriented in the y direction. Also, the control volume has unit depth in the z direction (perpendicular to the page). Hence, for these sections, dS = dy(1). The integral in Equation (6) becomes

$$\oiint_S (\rho \mathbf{V} \cdot \mathbf{dS})\, u = -\int_i^a \rho_i u^2{}_1 dy + \int_h^b \rho_2 u^2{}_2 dy \quad (7)$$

Note that the minus in front of the first term on the right-hand side of Equation (7) is due to **V** and **dS** being in opposite directions along ai (station 1 is an inflow boundary); in contrast, **V** and **dS** are in the same direction over hb (station 2 is an outflow boundary), and hence the second term has a positive sign.

Before going further with Equation (7), consider the integral form of the continuity equation for steady flow. Applied to the control volume in Figure A, this becomes

$$-\int_i^a \rho_1 u_1\, dy + \int_h^b \rho_2 u_2\, dy = 0$$

or,

$$\int_i^a \rho_1 u_1\, dy = \int_h^b \rho_2 u_2\, dy \quad (8)$$

Multiplying Equation (8) by u_1, which is a constant, we obtain

$$\int_i^a \rho_1 u^2{}_1 dy = \int_h^b \rho_2 u_2 u_1\, dy \quad (9)$$

Substituting Equation (9) into Equation (7), we have

$$\oiint_S (\rho \mathbf{V} \cdot \mathbf{dS})\, u = -\int_h^b \rho_2 u_2 u_1\, dy + \int_h^b \rho_2 u^2{}_2 dy$$

or

$$\oiint_S (\rho \mathbf{V} \cdot \mathbf{dS})\, u = - \int_h^b \rho_2 u_2 (u_1 - u_2)\, dy \qquad (10)$$

Substituting Equation (10) into Equation (6) yields

$$D' = \int_h^b \rho_2 u_2 (u_1 - u_2)\, dy \qquad (11)$$

Equation (11) is the desired result of this section; it expresses the drag of a body in terms of the known freestream velocity u_1 and the flow-field properties ρ_2 and u_2, across a vertical station downstream of the body. These downstream properties can be measured in a wind tunnel, and the drag per unit span of the body D' can be obtained by evaluating the integral in Equation (11) numerically, using the measured data for ρ_2 and u_2 as a function of y.

Examine Equation (11) more closely. The quantity $u_1 - u_2$ is the velocity decrement at a given y location. That is, because of the drag on the body, there is a wake that trails downstream of the body. In this wake, there is a loss in flow velocity $u_1 - u_2$. The quantity $\rho_2 u_2$ is simply the mass flux; when multiplied by $u_1 - u_2$, it gives the decrement in momentum. Therefore, the integral in Equation (11) is physically the decrement in momentum flow that exists across the wake, and from Equation (11), this wake momentum decrement is equal to the drag on the body.

For incompressible flow, ρ = constant and is known. For this case, Equation (11) becomes

$$D' = \rho \int_h^b u_2 (u_1 - u_2)\, dy \qquad (12)$$

Equation (12) is the answer to the questions posed at the beginning of this section. It shows how a measurement of the velocity distribution across the wake of a body can yield the drag. These velocity distributions are conventionally measured with a Pitot rake.

2.2

Denote the pressure distributions on the upper and lower walls by $p_u(x)$ and $p_\ell(x)$ respectively.

The walls are close enough to the model such that p_u and p_ℓ are not necessarily equal to p_∞.

Assume that faces <u>ai</u> and <u>bh</u> are far enough upstream and downstream of the model such that

$p = p_\infty$ and $v = 0$ and <u>ai</u> and <u>bh</u>.

Take the y-component of Eq. (2.11) in the text:

$$L = - \oiint_S (\rho \vec{V} \cdot \vec{dS})\ v - \iint_{abhi} (p\, \vec{dS})_y$$

The first integral = 0 over all surfaces, either because $\vec{V} \cdot \vec{ds} = 0$ or because $v = 0$. Hence

$$L' = - \iint_{abhi} (p\, \vec{dS})_y = - \left[\int_a^b p_u\, dx - \int_i^h p_\ell\, dx \right]$$

Minus sign because y-component is in downward direction.

Note: In the above, the integrals over <u>ia</u> and <u>bh</u> cancel because $p = p_\infty$ on both faces. Hence

$$L' = \int_i^h p_\ell\, dx - \int_a^b p_u\, dx$$