Solutions Manual©

to accompany

*System Dynamics*, Third Edition

by

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Solutions to Problems in Chapter One

**1.1** $W = mg = 3(32.2) = 96.6$ lb.

**1.2** $m = W/g = 100/9.81 = 10.19$ kg. $W = 100(0.2248) = 22.48$ lb. $m = 10.19(0.06852) = 0.698$ slug.

**1.3** $d = (50 + 5/12)(0.3048) = 15.37$ m.

**1.4** $d = 3(100)(0.3048) = 91.44$ m

**1.5** $d = 100(3.281) = 328.1$ ft

**1.6** $d = 50(3600)/5280 = 34.0909$ mph

**1.7** $v = 100(0.6214) = 62.14$ mph

**1.8** $n = 1/[60(1.341 \times 10^{-3})] = 12.43$, or approximately 12 bulbs.

**1.9** $5(70 - 32)/9 = 21.1°$ C

**1.10** $9(30)/5 + 32 = 86°$ F

**1.11** $\omega = 3000(2\pi)/60 = 314.16$ rad/sec. Period $P = 2\pi/\omega = 60/3000 = 1/50$ sec.

**1.12** $\omega = 5$ rad/sec. Period $P = 2\pi/\omega = 2\pi/5 = 1.257$ sec. Frequency $f = 1/P = 5/2\pi = 0.796$ Hz.

**1.13** Speed $= 40(5280)/3600 = 58.6667$ ft/sec. Frequency $= 58.6667/30 = 1.9556$ times per second.

**1.14** $x = 0.005 \sin 6t$, $\dot{x} = 0.005(6) \cos 6t = 0.03 \cos 6t$. Velocity amplitude is 0.03 m/s. $\ddot{x} = -6(0.03) \sin 6t = -0.18 \sin 6t$. Acceleration amplitude is 0.18 m/s$^2$. Displacement, velocity and acceleration all have the same frequency.

**1.15** Physical considerations require the model to pass through the origin, so we seek a model of the form $f = kx$. A plot of the data shows that a good line drawn by eye is given by $f = 0.2x$. So we estimate $k$ to be 0.2 lb/in.

**1.16** The script file is

```
x = [0:0.01:1];
subplot(2,2,1)
plot(x,sin(x),x,x),xlabel(′x (radians)′),ylabel(′x and sin(x)′),...
gtext(′x′),gtext(′sin(x)′)
subplot(2,2,2)
plot(x,sin(x)-x),xlabel(′x (radians)′),ylabel(′Error:  sin(x) - x′)
subplot(2,2,3)
plot(x,100*(sin(x)-x)./sin(x)),xlabel(′x (radians)′),...
ylabel(′Percent Error′),grid
```

The plots are shown in the figure.

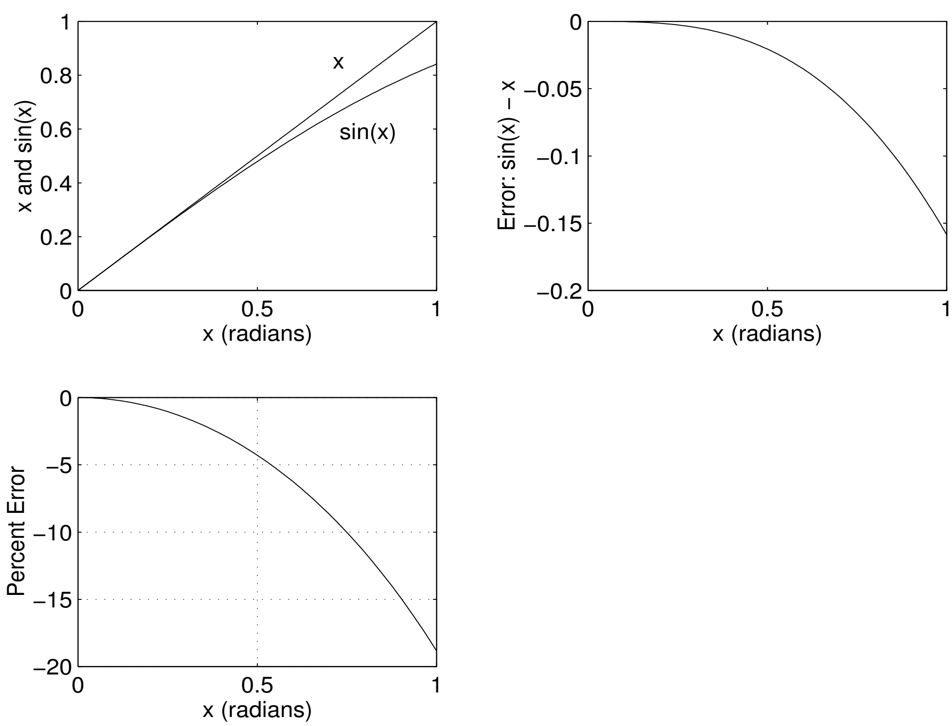


Figure : for Problem 1.16.

From the third plot we can see that the approximation $\sin x \approx x$ is accurate to within 5% if $|x| \leq 0.5$ radians.

**1.17** For $\theta$ near $\pi/4$,

$$f(\theta) \approx \sin\frac{\pi}{4} + \left(\cos\frac{\pi}{4}\right)\left(\theta - \frac{\pi}{4}\right)$$

For $\theta$ near $3\pi/4$,

$$f(\theta) \approx \sin\frac{3\pi}{4} + \left(\cos\frac{3\pi}{4}\right)\left(\theta - \frac{3\pi}{4}\right)$$

**1.18** For $\theta$ near $\pi/3$,

$$f(\theta) \approx \cos\frac{\pi}{3} - \left(\sin\frac{\pi}{3}\right)\left(\theta - \frac{\pi}{3}\right)$$

For $\theta$ near $2\pi/3$,

$$f(\theta) \approx \cos\frac{2\pi}{3} - \left(\sin\frac{2\pi}{3}\right)\left(\theta - \frac{2\pi}{3}\right)$$

**1.19** For $h$ near 25,

$$f(h) \approx \sqrt{25} + \frac{1}{2\sqrt{25}}(h-25) = 5 + \frac{1}{10}(h-25)$$

**1.20** For $r$ near 5,

$$f(r) \approx 5^2 + 2(5)(r-5) = 25 + 10(r-5)$$

For $r$ near 10,

$$f(r) \approx 10^2 + 2(10)(r-10) = 100 + 20(r-10)$$

**1.21** For $h$ near 16,

$$f(h) \approx \sqrt{16} + \frac{1}{2\sqrt{16}}(h-16) = 4 + \frac{1}{8}(h-16)$$

$f(h) \geq 0$ if $h > -16$.

**1.22** Construct a straight line the passes through the two endpoints at $p = 0$ and $p = 900$. At $p = 0$, $f(0) = 0$. At $p = 900$, $f(900) = 0.002\sqrt{900} = 0.06$. This straight line is

$$f(p) = \frac{0.06}{900}p = \frac{1}{15,000}p$$

**1.23** (a) The data is described approximately by the linear function $y = 54x - 1360$. The precise values given by the least squares method (Appendix C) are $y = 53.5x - 1354.5$.

(b) Only the loglog plot of the data gives something close to a straight line, so the data is best described by a power function $y = bx^m$ where the approximate values are $m = -0.98$ and $b = 3600$. The precise values given by the least squares method (Appendix C) are $y = 3582.1x^{-0.9764}$.

(c) Both the loglog and semilog plot (with the $y$ axis logarithmic) give something close to a straight line, but the semilog plot gives the straightest line, so the data is best described by a exponential function $y = b(10)^{mx}$ where the approximate values are $m = -0.007$ and $b = 2.1 \times 10^5$. The precise values given by the least squares method (Appendix C) are $y = 2.0622 \times 10^5(10)^{-0.0067x}$.

**1.24** With this problem, it is best to scale the data by letting $x = year - 2005$, to avoid raising large numbers like 2005 to a power. Both the loglog and semilog plot (with the $y$ axis logarithmic) give something close to a straight line, but the semilog plot gives the straightest line, so the data is best described by a exponential function $y = b(10)^{mx}$. The approximate values are $m = 0.035$ and $b = 9.98$.

Set $y = 20$ to determine how long it will take for the population to increase from 10 to 20 million. This gives $20 = 9.98(10)^{0.03x}$. Solve it for $x$: $x = (\log(20) - \log(9.98))/0.035$. The answer is 8.63 years, which corresponds to 8.63 years after 2005.

**1.25** (a) If $C(t)/C(0) = 0.5$ when $t = 500$ years, then $0.5 = e^{-5500b}$, which gives $b = -\ln(0.5)/5500 = 1.2603 \times 10^{-4}$.

(b) Solve for $t$ to obtain $t = -\ln[C(t)/C(0)]/b$ using $C(t)/C(0) = 0.9$ and $b = 1.2603 \times 10^{-4}$. The answer is 836 years. Thus the organism died 836 years ago.

(c) Using $b = 1.1(1.2603 \times 10^{-4})$ in $t = -\ln(0.9)/b$ gives 760 years. Using $b = 0.9(1.2603 \times 10^{-4})$ in $t = -\ln(0.9)/b$ gives 928 years.

**1.26** Only the semilog plot of the data gives something close to a straight line, so the data is best described by an exponential function $y = b(10)^{mx}$ where $y$ is the temperature in degrees C and $x$ is the time in seconds. The approximate values are $m = -3.67$ and $b = 356$. The alternate exponential form is $y = be^{(m \ln 10)x} = 356e^{-8.451x}$. The time constant is $1/8.451 = 0.1183$ s.

The precise values given by the least squares method (Appendix C) are $y = 356.0199(10)^{-3.6709x}$.

**1.27** Only the semilog plot of the data gives something close to a straight line, so the data is best described by an exponential function $y = b(10)^{mx}$ where $y$ is the bearing life thousands of hours and $x$ is the temperature in degrees F. The approximate values are $m = -0.007$ and $b = 142$. The bearing life at 150 ° F is estimated to be $y = 142(10)^{-0.007(150)} = 12.66$, or 12,600 hours. The alternate exponential form is $y = be^{(m \ln 10)x} = 142e^{-0.0161x}$. The time constant is $1/0.0161 = 62.1$ or $6.21 \times 10^4$ hr.

The precise values given by the least squares method (Appendix C) are $y = 141.8603(10)^{-0.0070x}$.

**1.28** Only the semilog plot of the data gives something close to a straight line, so the data is best described by an exponential function $y = b(10)^{mx}$ where $y$ is the voltage and $x$ is the time in seconds. The first data point does not lie close to the straight line on the semilog plot, but a measurement error of $\pm 1$ volt would account for the discrepancy. The approximate values are $m = -0.43$ and $b = 96$. The alternate exponential form is $y = be^{(m \ln 10)x} = 96e^{-0.99x}$. The time constant is $1/0.99 = 1.01$ s.

The precise values given by the least squares method (Appendix C) are $y = 95.8063(10)^{-0.4333x}$.

**1.29** A semilog plot generated by the following script file shows that the exponential function $T - 70 = be^{mt}$ fits the data well.

```
t = [0:300:3000];
temp = [207,182,167,155,143,135,128,123,118,114,109];
DT = temp-70;
semilogy(t,DT,t,DT,'o')
```

Fitting a line by eye gives the approximate values $m = -4 \times 10^{-4}$ and $b = 125$. The corresponding function is $T(t) = 70 + 125e^{-4\times10^{-4}t}$.

The precise values given by the least squares method (Appendix C) are $m = -4.0317 \times 10^{-4}$ and $b = 125.1276$.

**1.30** Plots of the data on a log-log plot and rectilinear scales both give something close to a straight line, so we try both functions. (Note that the flow should be 0 when the height is 0, so we do not consider the exponential function and we must force the linear function to pass through the origin by setting $b = 0$.) The three lowest heights give the same time, so we discard the heights of 1 and 2 cm.

The power function fitted by eye in terms of the height $h$ is approximately $f = 4h^{0.9}$. Note that the exponent is not close to 0.5, as it is for orifice flow. This is because the flow through the outlet is pipe flow. For the linear function $f = mh$, the best fit by eye is approximately $f = 3.2h$.

Using the least squares method (Appendix C) gives more precise results: $f = 4.1595h^{0.8745}$ and $f = 3.2028h$.

**1.31** Plots of the data on a log-log plot and rectilinear scales both give something close to a straight line, so we try both functions. (Note that the flow should be 0 when the height is 0, so we do not consider the exponential function and we must force the linear function to pass through the origin by setting $b = 0$.) The variable `x` is the height and the variable `y` is the flow rate. The three lowest heights give the same time, so we discard the heights of 1 and 2 cm.

The power function fitted by eye in terms of the height $h$ is approximately $f = 4h^{0.9}$. Note that the exponent is not close to 0.5, as it is for orifice flow. This is because the flow through the outlet is pipe flow. For the linear function $f = mh$, the best fit by eye is approximately $f = 3.7h$.

Using the least squares method (Appendix C) gives more precise results: $f = 4.1796h^{0.9381}$ and $f = 3.6735h$.